Algebra Study Sheet

Basic Properties and Identities

Arithmetic and Fraction Operations

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$
$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$
$$\frac{a+b}{c} \neq \frac{a}{c} + \frac{b}{c} \text{ unless simplified}$$

Exponent Rules

$$a^m \cdot a^n = a^{m+n}$$
$$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$
$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n$$
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$$
$$a^0 = 1, \quad a \neq 0$$
$$a^{-n} = \frac{1}{a^n}$$

Radical Properties

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \quad b \neq 0$$
$$\sqrt{a^2} = |a|$$

Factoring and Solving Equations

$$x^2 - a^2 = (x - a)(x + a)$$
$$x^2 + 2ax + a^2 = (x + a)^2$$
$$x^2 - 2ax + a^2 = (x - a)^2$$
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$
$$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$$
$$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$$
$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x^2 = p \Rightarrow x = \pm\sqrt{p}$$

Functions and Graphs

$f(x) = a$ is a horizontal line at $y = a$

$f(x) = mx + b$ has slope m and intercept b

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{(slope)}$$
$$y - y_1 = m(x - x_1) \quad \text{(point-slope form)}$$

$f(x) = ax^2 + bx + c$ has vertex at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{(circle)}$$
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \quad \text{(ellipse)}$$
$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \quad \text{(hyperbola, horizontal)}$$
$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1 \quad \text{(hyperbola, vertical)}$$

Common Algebraic Mistakes

- Division by zero is undefined: $\frac{1}{0}$ is not a number.
- $\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a+b}$
- $\sqrt{9 + 16} \neq \sqrt{9} + \sqrt{16}$
- $(x + a)^2 \neq x^2 + a^2$
- $(a + b)^2 = a^2 + 2ab + b^2$
- $\frac{a+b}{c} \neq \frac{a}{c} + \frac{b}{c}$ unless simplified

Inequality Properties

$$a < b \Rightarrow a + c < b + c \text{ and } a - c < b - c$$
$$a < b,\ c > 0 \Rightarrow ac < bc$$
$$a < b,\ c < 0 \Rightarrow ac > bc$$

Absolute Value

$$|a| = \begin{cases} a & a \geq 0 \\ -a & a < 0 \end{cases} \quad \text{and} \quad |a + b| \leq |a| + |b|$$

Distance Formula

For points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$,

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i = \sqrt{-1}, \quad i^2 = -1$$
$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) - (c + di) = (a - c) + (b - d)i$$
$$(a + bi)(c + di) = ac - bd + (ad + bc)i$$
$$\overline{a + bi} = a - bi$$
$$|a + bi| = \sqrt{a^2 + b^2}$$

Logarithms

$$\log_b x = y \iff b^y = x$$
$$\log x = \log_{10} x, \quad \ln x = \log_e x, \quad e \approx 2.718$$
$$\log_b 1 = 0, \quad \log_b b = 1$$
$$\log_b(xy) = \log_b x + \log_b y$$
$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$
$$\log_b(x^r) = r \log_b x, \quad \text{Domain: } x > 0$$

- $-(a+b) \neq -a+b$, instead: $-(a+b) = -a-b$
- $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$
- $\frac{a+b}{a} \neq 1+b$
- $x^a + x^b \neq x^{a+b}$
- $(a+b)^n \neq a^n + b^n$ unless $n = 1$

This sheet summarizes key identities and cautions against common errors. Use it for review, problem solving, or exam prep.